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AN APPROXIMATE METHOD FOR THE ANALYSIS OF RIGID FRAMED MEDIUM RISE BUILDING SUBJECTED TO AERODYNAMIC LOADINGS

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ABSTRACT

The investigation was an approximate method for the analysis of rigid framed medium rise building subjected to aerodynamic loadings. The main aim of the research work is to develop appropriate equations and formulae that will enable the use of the building variable dimension (i.e. such as L_1, L_2, h etc.) and either table calculators or other related computer spread sheet to determine the axial, shearing forces and their corresponding bending moments on the columns and beams; at any level of a rigid unbraced frame subjected mainly to aerodynamic loadings. It assumed that at the middle of columns and beams, are located points of contra flexures. The axial forces caused by the effects of the lateral loadings in the columns are proportional to their distances from the center of gravity of the frame. For low and medium rise buildings, the columns are assumed to be of the same cross-sectional area. For buildings lower than five storeys, wind loadings are negligible since the floor slab or diaphragm is assumed to be infinitely rigid but for those higher than five storeys, the effects of wind loadings are considered important for the analysis of the structural system. At heights greater than 10m, the wind speed is assumed to increase approximately, linearly with height. Therefore, a coefficient k, was incorporated to take into consideration the increase of the basic wind speed with height, local site conditions, building category, directionality and height coefficient respectively. Useful formulae and results obtained show the approaches are appropriate methods that can be used to obtain approximate preliminary sections suitable for analysis and design of beams and columns for low and medium rise multi-storey building subjected to wind gust anywhere in the World.

KEYWORDS:- Aerodynamic loadings, rigid frame, multi-storey building, wind speed.

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INTRODUCTION

Rigid Frame Multi-Storey Buildings and Methods of Analysis

Rigid-frame high-rise buildings typically consist of a conglomeration of parallel or orthogonally arranged structural elements such as columns and beams or girder with moment resisting joints. Resistance to horizontal loadings is usually provided by the flexural resistance of the slab diaphragms, columns, girders and the monolithic joints. The continuity of the frame also contributes to the resistance from the vertical loadings such as dead and superimposed live loads by reducing the bending moment in the beams or girders. Simplified practical examples of this method of design for columns have been elaborately discussed by many authors (Oyenuga 2011, Mosley *et al.;*2007)

According to Smith and Coull (1991), rigid frame is economical only for buildings up to about 25 stories; above 25 stories, the relatively high lateral flexibility of the frame will require very uneconomically large members for the

drift at higher floors to be effectively controlled. Gravity loading is also resisted by the rigid-frame action (Oyenuga 2011). Negative moments are induced in the girders adjacent to the column causing the midspans positive moments to be significantly reduced when compared with midspans moments from equivalent span of a simply supported beam. Regardless of this advantage, in structures where gravity loads dictate the design concept, economies that arise from this effect tend to be offset by the high cost required to actualize effectively monolithic rigid joints.

According to Grider *et al.;* (1999), a number of methods have been developed over the years for the analysis of continuous beams and frames. The so-called classical methods—such as application of the theorem of three moments, the method of least work, and the general method of consistent deformation have proved useful mainly in the analysis of continuous beams having few spans or of very simple frames. For the more complicated cases usually met in practice, such methods prove to be exceedingly tedious, and alternative approaches are preferred. For many years the closely related methods of slope deflection and moment distribution provided the basic analytical tools for the analysis of indeterminate beams and frames. In offices with access to high-speed digital computers, these have been supplanted largely by matrix methods of analysis. Where computer facilities are not available, moment distribution is still the most common method. Approximate methods of analysis, based either on an assumed shape of the deformed structure or on moment coefficients, provide a means for rapid estimation of internal forces and moments. Such estimates are useful in preliminary design and in checking more exact solutions and in structures of minor importance may serve as the basis for final design.

The method of slope deflection entails writing two equations for each member of a continuous frame, one at each end, expressing the end moment as the sum of four contributions: (1) the restraining moment associated with an assumed fixed-end condition for the loaded span, (2) the moment associated with rotation of the tangent to the elastic curve at the near end of the member; the moment associated with rotation of the tangent at the far end of the member, and the moment associated with translation of one end of the member with respect to the other. These equations are related through application of requirements of equilibrium and compatibility at the joints. A set of simultaneous, linear algebraic equations results for the entire structure, in which the structural displacements are unknowns. Solution for these displacements permits the calculation of all internal forces and moments. This method is well suited to solving continuous beams, provided there are not very many spans. Its usefulness is extended through modifications that take advantage of symmetry and anti-symmetry and of hinge-end support conditions where they exist. However, for multistory and multi-bay frames in which there are a large number of members and joints, and which will, in general, involve translation as well as rotation of these joints, the effort required to solve the correspondingly large number of simultaneous equations is prohibitive (Grider *et al.;*1999).

Use of matrix theory makes it possible to reduce the detailed numerical operations required in the analysis of an indeterminate structure to systematic processes of matrix manipulation which can be performed automatically and rapidly by computer. Such methods permit the rapid solution of problems involving large numbers of unknowns. As a consequence, less reliance is placed on special techniques limited to certain types of problems; powerful methods of general applicability have emerged, such as the matrix displacement method. Account can be taken of such factors as rotational restraint provided by members perpendicular to the plane of a frame. A large number of alternative loadings may be considered. Provided that computer facilities are available, highly precise analyses are possible at lower cost than for approximate analyses previously employed.

In spite of the development of refined methods for the analysis of beams and frames, increasing attention is being paid to various approximate methods of analysis. There are several reasons for this. Prior to performing a complete analysis of an indeterminate structure, it is necessary to estimate the proportions of its members in order to know their relative stiffness upon which the analysis depends. These dimensions can be obtained using approximate analysis. Also, even with the availability of computers, most engineers find it desirable to make a rough check of results using approximate means to detect gross errors. Further, for structures of minor importance, it is often satisfactory to design on the basis of results obtained by rough calculation. Provided that points of inflection

(locations in members at which the bending moment is zero and there is a reversal of curvature of the elastic curve) can be located accurately, the stress resultants for a framed structure can usually be found on the basis of static equilibrium alone. Each portion of the structure must be in equilibrium under the application of its external loads and the internal stress resultants.

Most of the methods mentioned above involve many hours of rigorous analysis and if care is not taken, inherent mistakes and errors could creep into the computational process. Even the use of computer methods and soft ware's on most occasions require the entry of large volumes of input and output data or the development of complex algorithms for solutions of multi-storey framed buildings. But for reliable solutions to be obtained, preliminary sizing of main structural members must be determined by approximate methods and entered as input. Therefore, the main aim of this research work is to develop appropriate equations and formulae that will enable the use of the building variable dimension (i.e. such as L_1, L_2, h etc.) and either table calculators or other related computer spread sheet to determine the axial, shearing forces and their corresponding bending moments on the columns and beams; at any level of a rigid unbraced frame subjected mainly to aerodynamic loadings. These formulae are mainly applicable to the full or preliminary assessment of structural members for: 1) For full assessment of medium rise multi-storey buildings that satisfies height / width ratio less than six, overall height less than 50m and subjected mainly to aerodynamic loadings. 2.) They are also useful for the preliminary assessment of buildings that have dynamic response with minimum frequency ($f < 1.0$ Hz) or fundamental natural period exceeding one second ($T > 1$ sec) (Onundi *et al.;* 2012).

Wind and Wind Loadings

Taranath (2005), wind is a term used to describe horizontal motion of air. Motion in a vertical direction is called a current. Winds are produced by differences in atmospheric pressure that are primarily attributable to differences in temperature. These differences are caused largely by unequal distribution of heat from the sun, and the difference in thermal properties of land and ocean surfaces. When temperatures of adjacent regions become unequal, the warmer, lighter air rises and flows over the colder, heavier air. Winds initiated in this way are modified by rotation of the earth. Near the equator, the lower atmosphere is warmed by the sun's heat. Forces due to wind are generated on the exterior of the building based on its height, local ground surface roughness (hills, trees, and other buildings) and the square of the wind velocity. The weight of the building, unlike in earthquake design, has little effect on wind forces, but is helpful in resisting uplift forces. Unless the structure has large openings, all the wind forces are applied to the exterior surfaces of the building. This is in contrast to earthquake forces where both exterior and interior walls are loaded proportionally to their weight. Wind pressures act inward on the windward side of a building and outward on most other sides and most roof surfaces. Special concentrations of outward force, due to aerodynamic lift, occur at building corners and roof edges, particularly so at overhangs or eaves of the building. The overall structure is designed for the sum of all lateral and uplift pressures and the individual parts to resist the outward and inward pressure concentrations. They must be connected to supporting members to form a continuous load path.

According to Kijewski and Kareem (2001), Mendis *et al.;*(2007) and Joseph and William (2006), many aspects involved in the estimation of wind loads are held in common by the various international codes and standards. Instead of commenting on them repeatedly, they have been highlighted in their work briefly that::

a) All the standards, subdivided the global terrain into 3 to 5 categories depending on how they affect the wind characteristics at that location.
b) The design wind speed, associated with one or a range of mean recurrence intervals, used in analysis by each of the codes is typically the product of the basic wind speed and factors to account for the geographic location, topographical effects, building size and surface roughness, etc.
c) Wind gustiness introduces dynamic load effects which the codes and standards account for by factoring up the mean loads by a gust factor. Both time and spatial averaging play an important role in the development of gust factors. For a very small size structure, a short duration gust, which

completely engulfs the structure, e.g. a 3 second gust, may be adequate to account for the effects of gustiness, in which case the gust factor is unity.

d) On the other hand, if the wind-averaging interval is higher, e.g. 10 minutes or more, the averaged wind exhibits less fluctuation, and accordingly the gust factor is greater than unity. This departure from unity is affected not only by the averaging interval, but also by the site terrain and the size and dynamic characteristics of the structure.

e) Furthermore, it should also be noted that while all of the standards reference their wind speed at 10 m above ground in a flat, open exposure, each uses gusts of different duration. The British and Canadian standards use the mean hourly wind speed in design, while the European Pre standard, the China National Standard and the AIJ Recommendations all use a 10-minute mean wind velocity. The ASCE7-95 standard references a 3 second gust, as does the Australian Standard, though, in the latter case. This wind is later converted to a mean hourly wind for subsequent calculations of dynamic pressure and the gust factor. As a result, for any adequate comparison amongst standards, there must be proper adjustments of the reference velocity.

Determination of the aerodynamic loading on the Building

The dynamic wind load, F along the height, L of a high rise building is given as

$$F_1 = P_e A \qquad \text{…... (1)}$$

Where;

Pe = the pressure acting on the external surface of a building

$P_e = q_s C_{pe} C_a C_r$ (BS 6399 (2004) …….(1a)

C_{pe} = the external pressure coefficient for the building surface given in clauses 2.4 and 2.5 of the Code; C_a = the size effect factor for external pressures defined in clause 2.1.3.4.

q_s = the dynamic pressure from clause 2.1.2 and q_s given as $0.613V_e^2$

V_e = the effective average basic wind speed in m / sec (Soboyejo, 1971, Onundi *et al.;* 2009 and Onundi, 2010) and

A = the loaded area

$$\therefore F_1 = q_s A C_{pe} C_a C_r \qquad \text{…….(1b)}$$

The dynamic augmentation factor, (C_r) recommended by the Clause 1.6.1, Figure 3; BS 6399 (2004) for the high rise building. The lateral loadings acting on a structure may be caused by either wind pressures, earthquakes (i.e. seismic forces) or earth retained by the walls of the structure. The method described in this study assumes that the vertical loadings are computed separately and will be combined with the results obtained from the effects of the lateral loadings.

Analysis of Medium Rise Buildings

For the design of low to medium high rise buildings, the elastic method of analysis and the following approximation methods of analyses may be assumed (Mosley *et al.;* 2011).

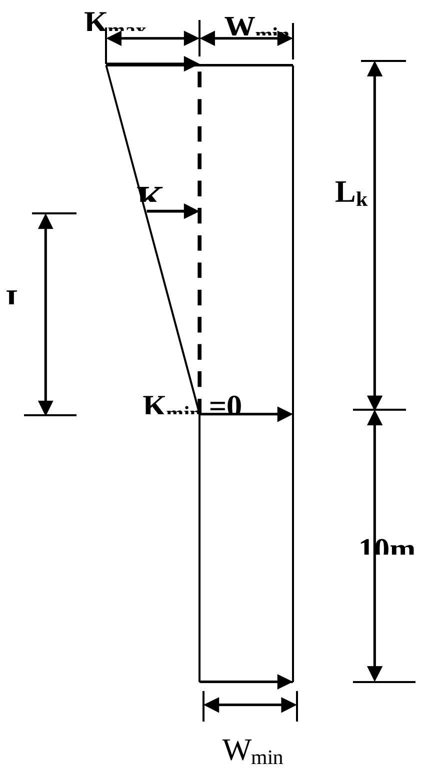
i. At the middle of columns and beams, are located points of contra flexures.

ii. The axial forces caused by the effects of the lateral loadings in the columns are proportional to their distances from the center of gravity of the frame. For low and medium rise buildings, the columns are assumed to be of the same cross-sectional area.

iii. For buildings lower than five storeys, wind loadings are negligible since the floor slab or diaphragm is assumed to be infinitely rigid but for those higher than five storeys, the effects of wind loadings are considered important for the analysis of the structural system. At heights greater than 10m, the wind speed

is assumed to increase approximately, linearly with height. Therefore, a coefficient k, is incorporated to take into consideration the increase of the basic wind speed with height, local site conditions, building category, directionality and height coefficient respectively as shown in Figure 1.0(a).

By using theprinciple of similar triangle, it becomes possible for intermediate values, k_x between k_{max} and k_{min} to be determined from the following eq



$$\frac{K_x}{L_x} = \frac{K_{max}}{L_k}$$

Therefore,

$$K_x = K_{max}\frac{L_x}{L_k} \qquad \text{......(2)}$$

If $L_x = 0$;

$k_x = k_{min} = k = 0$

and $W_{min} = k + W = W$. ......(2a)

This will correspond to the basic wind speed W, at 10m height for the site.

If $0 < L_x < L_k$,

$k_x = k_{int}$

and $W_{int} = k_x + W_{min} = k_{int} + W_{min}$. ......(2b)

This will correspond to the wind speed intermediate between 10m and the maximum height L_k, of the building.

If $L_x = L_k$,

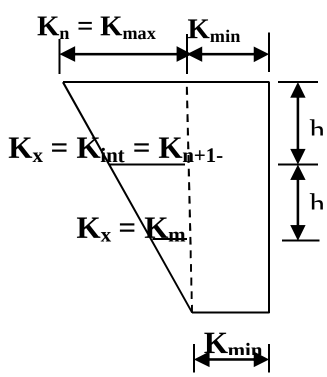
$k_x = k_{max}$and

$Wmax = k_x + W_{min} = k_{max} + W_{min}$. ......(2c)

This will correspond to the maximum wind speed corresponding to the maximum height of the building.

Figure 1.0 (a) : Linear Variation of Wind Speed along the Building

a.) At the Top Floors



To determine the nodal force F_i at any storey height i, the following Procedures are suggested.

Let the force at top floor be represented by $\mathbf{F_n}$ and W_{max} be the expected maximum wind pressure at the building top floor Figure1.0(b) The corresponding wind pressure at the building intermediate storey heights between the top floor and 10m height measured from ground level and is represented by W_{int} and the nodal forces are also determined by F_{n-m}.

Figure 1.0(b) : Linear Variation of Wind Speed above 10m Height

Let $W_{max} = (K_{max} + W_{min})b$
$W_{int} = (K_{int} + W_{min})b$
But

$$\frac{K_{xj}}{jh} = \frac{K_{max}}{nh}; \cong K_{xj} = \frac{jh}{nh} K_{max}$$

$K_{xj} = K_j = \frac{j}{n} K_{max} = \frac{n+3}{n} K_{max}$ ……(2d)
n = total number of storeys in the multi-storey building
j = total number of storeys above the datum level (i.e. in this case, the datum is 10m)
b = Width or frame spacing

Therefore,
$W_j = \left(W_{min} + \frac{j}{n} K_{max}\right) b$
or
$W_j = (W_{min} + \frac{n+3}{n} K_{max})b$ ……(2e)
W_j = Wind speed at j^{th} storey level

Alternatively, for all the floors, when n > j > 10m
$K'_n = K'_m = K_{min} + K_n = K_{min} + \frac{n+3}{n} K_{max}$ and
$W_n = W_{max} = (W_{min} + \frac{n+3}{n} K_{max})b = 0.0.1042 \text{ b } W_{bas} \left(\frac{2n-3}{n-3}\right)$ ……(2f)
W_{bas} = Basic wind speed at the building site.

For the floors lower than 10m, when j ≤ 10m

$$K'_m = K_{min} + K_n = 0 + K_n$$
$$W_j = W_n = bW_{min}$$

……(2g)

Therefore,
Figure 2.0(a, b,c and d), shows that at the top floor, the nodal force is
$F_n = W\frac{h^2}{2}\left(\frac{K_{max}+K_{int}+2K_{min}}{2}\right) = W\frac{h^2}{4}(K_n + K_m + 2K_{min}) = K'_n W\frac{h^2}{4} = W_n\frac{h^2}{4}$
Whereas, the corresponding wind pressure at the building intermediate storey heights (i.e. floors) between the top floor and the five storey height have the nodal forces represented by F_{n+1-m}

$$F_{n-m} = W\frac{h^2}{2}\left(\frac{K_{n-m} + K_m + 2K_{min}}{2}\right) = W\frac{h^2}{4}(K_{n-m} + K_m + 2K_{min}) = K'_{n-m}W\frac{h^2}{4} = K'_{n-m}W_{min}\frac{h^2}{4} = W_j\frac{h^2}{4}$$

$F_m = W\frac{h^2}{2}\left(\frac{K_m+2K_{min}}{2}\right) = W\frac{h^2}{4}(K_{n-m} + K_m + 2K_{min}) = K'_m W\frac{h^2}{4} = K'_m W_{min}\frac{h^2}{4} = W_m\frac{h^2}{4}$ ….(3)

Where
K_n = the maximum variable wind pressure coefficient corresponding to the building top floor (e.g. if n=10, hence $K_n = K_{10}$)
K_{n-m} = the intermediate variable wind pressure coefficient corresponding to the building intermediate floors counted from the top (e.g. if n=10, m=8, hence $K_{n-m} = K_{10-8} = K_2$). This is the second to the topmost floor or in order words second floor from the top.
K_m = the variable wind pressure coefficient corresponding to the next floor (counted from ground floor) to the intermediate floor the building being considered (e.g. if m=8, hence $K_m = K_8$).

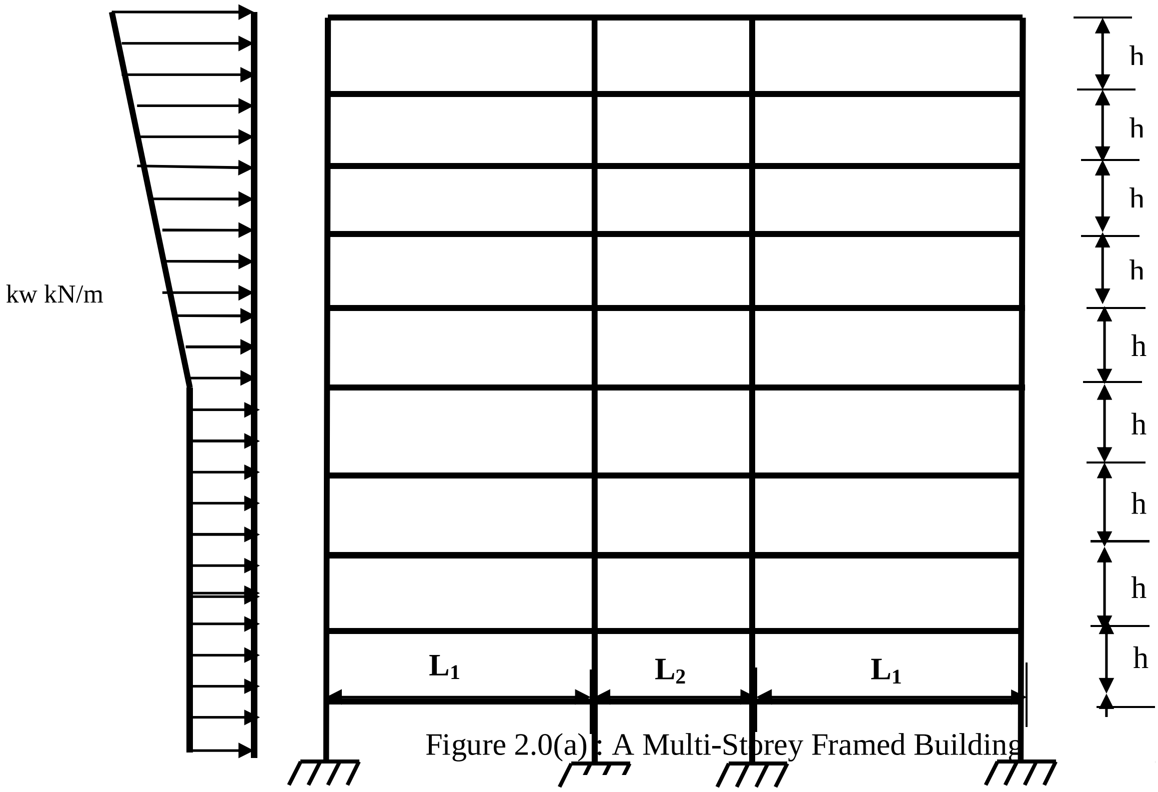


Figure 2.0(a) : A Multi-Storey Framed Building

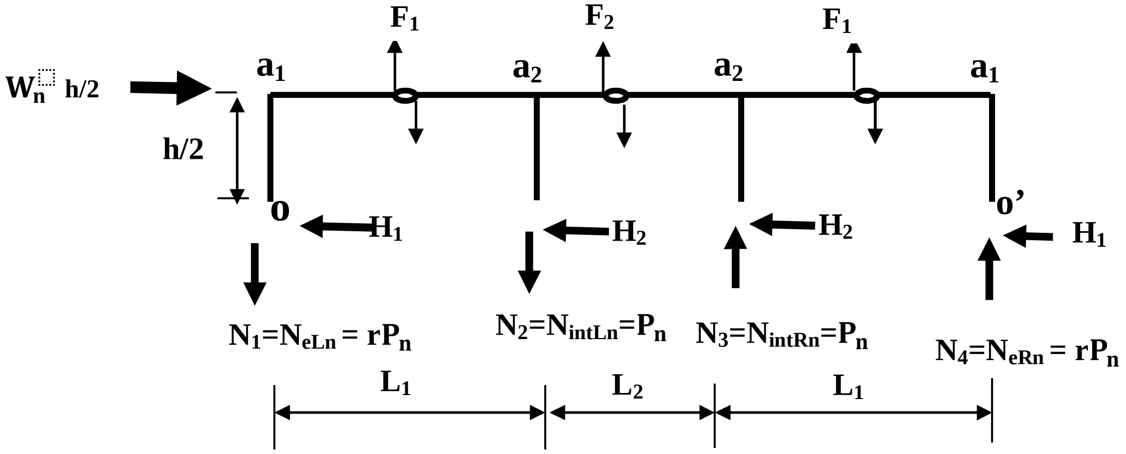


Figure 2.0 (b) : A Section through the Topmost Level of a Multi-Storey Framed

b.) The forces in columns at the top floor

The axial forces at the exterior columns (i.e. L= left and R=right) at the top floor are

$$N_1 = N_{eLn} = -N_{eRn} = -N_4 = r\,P_{1n} = rP$$

The axial forces at the interior columns (i.e. L= left and R=right) at the top floor are

$N_2 = N_{intLn} = -N_{intRn} = -N_3 = P_{2n} = P$ …..(4)

Where,

$P_{1n} = P_{2n} = P$ …..(4a)

$r = \frac{2(L_1+0.5L_2)}{L_2}$ …..(4b)

r = the proportional ratio between the columns and the center of gravity of the building frame.

Taking moments about point o;(i.e.; $\sum M_o = 0$)

$$W_n \frac{h}{2}\frac{h}{2} + PL_1 - (L_1 + L_2)P - (L_1 + L_2 + L_1)rP = 0$$

$$W_n \frac{h^2}{4} - P[2rL_1 + (r+1)\,L_2] = 0$$

Hence,

$P = \frac{W_n h^2}{4(2rL_1+(r+1)\,L_2)}$ …..(4c)

Since $N_1 = -N_4 = r\,P$

This is the axial force in the exterior columns at the top most floorof the building.

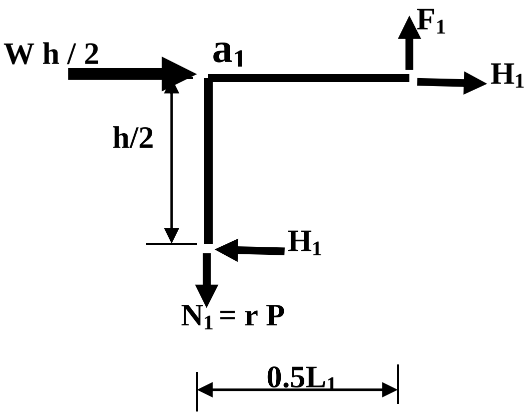
$N_1 = -N_4 = \frac{r\,W_n h^2}{4(2rL_1+(r+1)\,L_2)}$

and

$N_2 = -N_3 = P$

$N_2 = -N_3 = \frac{W_n h^2}{4(2rL_1+(r+1)\,L_2)}$ …..(4d)

Figure 2.0 (c), shows the horizontal force H_1 is determined by taking moments of all internal forces at joint a_1



$$\sum M_{a1} = 0$$

$$H_1 \frac{h}{2} - N_1 \frac{L_1}{2} = 0$$

$$H_1 = \left[N_1 \frac{L_1}{2}\frac{2}{h}\right] = N_1 \frac{L_1}{h}$$

Figure 2.0 (c) : A Section through the Topmost Level of a Multi-Storey Framed Continues

Since $N_1 = -N_4 = \frac{r\,W_n h^2}{4(2rL_1+(r+1)\,L_2)} = F_1$ …..(4e)

Therefore,

$$H_1 = \left[\left(\frac{rW_n h^2}{4(2rL_1+(r+1)\,L_2)}\right)\frac{L_1}{h}\right]$$

and

$$H_1 = \left[\left(\frac{r\,W_n L_1 h}{4(2rL_1+(r+1)\,L_2)}\right)\right]$$

$F_2 =$ the shear force on the beam along the exterior span of the rigid frame

Similarly, to determine $\mathbf{H_2}$ and $\mathbf{F_2}$; take sum moment about C_2 .

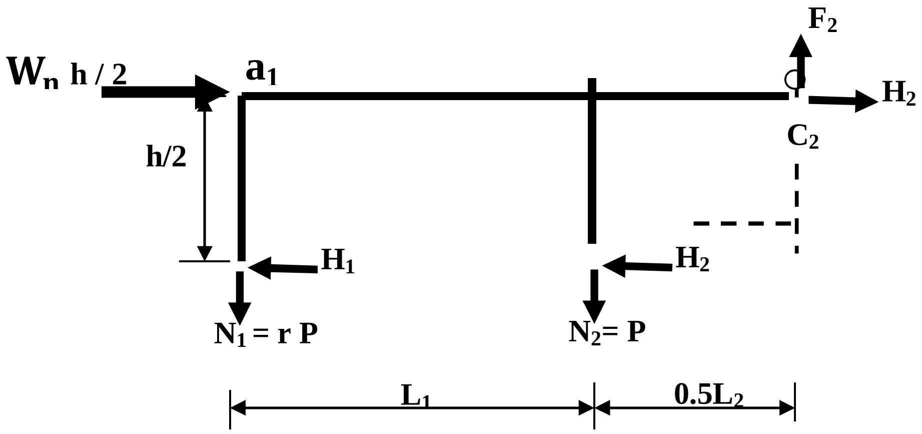


Figure 2.0 (d) : A Section through the Topmost Level of a Multi-Storey Framed Continues

Therefore,

$$\left(\text{i.e.}; \sum M_{C_2} = 0\right)$$

$$(H_1 + H_2)\frac{h}{2} - N_1\left(L_1 + \frac{L_2}{2}\right) - N_2\frac{L_2}{2} = 0$$

From this equation, H_2 is the only unknown variable and it can be determined

$$H_2 = \frac{2N_1}{h}\left(L_1 + \frac{L_2}{2}\right) + N_2\frac{L_2}{h} - H_1$$

Since,

$F_1 = N_1 = - N_4 = \frac{r\,W_n h^2}{4(2rL_1+(r+1)\,L_2)}$; $N_2 = - N_3 = \frac{W_n h^2}{4(2rL_1+(r+1)\,L_2)}$ and

$$H_1 = \left[\left(\frac{rW_n L_1 h}{4(2rL_1+(r+1)\,L_2)}\right)\right] \quad \text{.....(5)}$$

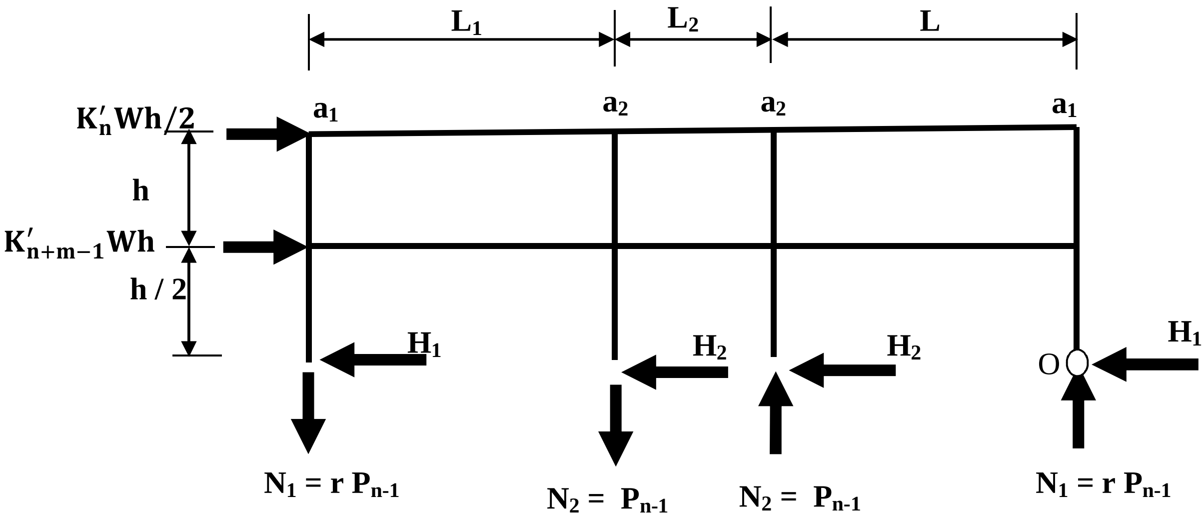
Substituting the values of N_1, N_2 and H_1; gives

$$H_2 = \left[\frac{r\,W_n h^2}{4(2rL_1+(r+1)\,L_2)}\right]\frac{2}{h}\left(L_1 + \frac{L_2}{2}\right) + \left[\frac{W_n h^2}{4(2rL_1+(r+1)\,L_2)}\right]\frac{L_2}{h} - \left[\frac{rW_n L_1 h}{4(2rL_1+(r+1)\,L_2)}\right]$$

$$H_2 = \frac{W_n h}{(2rL_1+(r+1)\,L_2)}\left[\frac{2r}{4}\left(L_1 + \frac{L_2}{2}\right) + \frac{L_2}{4} + \frac{rL_1}{4}\right]$$

$$H_2 = \frac{W_n h}{4(2rL_1+(r+1)\,L_2)}\left[rL_1 + (1+r)L_2\right] \quad \text{.....(5a)}$$

c.) Determination of Axial Force P_{n-1} at the Second to the Last Floor



From Figure 3.0: A Section through the Second Floor from the Top of a Multi-Storey Framed Building

$$\left(\text{i.e.}; \sum M_o = 0\right)$$

For convenience let's assume $K_{n-m} = k'$

$$\frac{k'_n W h_{n-1}}{2}\left(h_n + \frac{h_{n-1}}{2}\right) + k'W\left(\frac{h_n}{2} + \frac{h_{n-1}}{2}\right)\left(\frac{h_{n-1}}{2}\right) + L_1P_{n-1} - (L_1 + L_2)P_{n-1} - (L_1 + L_2 + L_1)rP_{n-1} = 0$$

$$\frac{k'_n W h_{n-1}}{2}h_n + \frac{k'_n W h_{n-1}}{2}\left(\frac{h_{n-1}}{2}\right) + k'W\left(\frac{h_n}{2}\frac{h_{n-1}}{2}\right) + k'W\left(\frac{h_{n-1}}{2}\right)^2 + L_1P_{n-1} - (L_1 + L_2)P_{n-1} - (L_1 + L_2 + L_1)rP_{n-1} = 0$$

If $h_n = h_{n-1} = h$; these equations will give

$$\frac{k'_n W h^2}{2} + \frac{k'_n W h^2}{4} + k'W\frac{h^2}{4} + k'W\frac{h^2}{4} + L_1P_{n-1} - (L_1 + L_2)P_{n-1} - (L_1 + L_2 + L_1)rP_{n-1} = 0$$

$$\frac{Wh^2}{2}\left(\frac{3k'_n}{2} + k'\right) + L_1P_{n-1} - L_1P_{n-1} - L_2P_{n-1} - (L_2 + 2L_1)rP_{n-1} = 0$$

$$\frac{Wh^2}{2}\left(\frac{3k'_n}{2} + k'\right) - P_{n-1}[L_2(1 + r) + 2rL_1] = 0$$

$$P_{n-1} = \frac{Wh^2\left(\frac{3k'_n}{2} + k'\right)}{2[L_2(1+r) + 2rL_1]} = \frac{Wh^2(3k'_n + 2k')}{4[L_2(1+r) + 2rL_1]}$$

Finally,

$$P_{n-1} = \frac{Wh^2(3k'_n + 2k')}{4[L_2(1+r) + 2rL_1]} = N_2 \quad \text{.....(6)}$$

To determine the internal forces at this pre-determined level, take moments along the beam and column at joints a_1 and a_2 respectively.

d.) At Joint a_1

Along the column

$$M_{a1} = H_1 \frac{h}{2}$$

Since

$$H_1 = \left[\left(\frac{r\,k_n' L_1 Wh}{4(2rL_1 + (r+1)\,L_2)}\right)\right]$$

Therefore,

$$M_{a1} = \left[\left(\frac{r\,k_n' L_1 Wh}{4(2rL_1 + (r+1)\,L_2)}\right)\right]\frac{h}{2}$$

and

$$M_{a1} = \left[\left(\frac{r\,k_n' L_1 Wh^2}{8(2rL_1 + (r+1)\,L_2)}\right)\right]$$

Along the beam L_1.

$$M_{a1} = F_1 \frac{L_1}{2}$$

$$F_1 = N_1 = -N_4 = \frac{r\,\mathbf{k_n'}\,Wh^2}{4(2rL_1 + (r+1)\,L_2)}$$

$$M_{a1} = \left[\left(\frac{r\,k_n' L_1 Wh^2}{8(2rL_1 + (r+1)\,L_2)}\right)\right] \qquad \text{.....(7)}$$

Similarly,

e.) At Joint a_2

Along the beam L_1

$$M_{a2_{L1}} = F_1 \frac{L_1}{2}$$

$$M_{a2_{L1}} = F_1 \frac{L_2}{2} = \left[\frac{r\,k_n'\,Wh^2}{4(2rL_1 + (r+1)\,L_2)}\right]\frac{L_2}{2} = \left[\frac{r\,k_n' L_1\,Wh^2}{8(2rL_1 + (r+1)\,L_2)}\right]$$

$$M_{a2_{L1}} = \left[\frac{rk_n' L_1 Wh^2}{8(2rL_1 + (r+1)\,L_2)}\right] \qquad \text{.....(7a)}$$

i.) Along the column h.

$$M_{a2_h} = H_2 \frac{h}{2}$$

$$H_2 = \frac{k_n' Wh}{4(2rL_1 + (r+1)\,L_2)}[rL_1 + (1+r)L_2]$$

$$M_{a2_h} = \frac{h}{2}\left\{\frac{k_n' Wh}{4(2rL_1 + (r+1)\,L_2)}[rL_1 + (1+r)L_2]\right\}$$

$$M_{a2_h} = \left\{\frac{k_n' Wh^2}{8(2rL_1 + (r+1)\,L_2)}[rL_1 + (1+r)L_2]\right\} \qquad \text{.....(7b)}$$

ii.) Along the beam L_2

$M_{a2_{L2}} = F_2 \frac{L_2}{2}$

But $F_2 = N_1 + N_2$

$N_1 = - N_4 = \frac{rk'_n Wh^2}{4(2rL_1+(r+1)\,L_2)} = rP$

and

$N_2 = - N_3 = P$

$N_2 = - N_3 = \frac{k'_n Wh^2}{4(2rL_1+(r+1)\,L_2)} = P$

$$F_2 = \frac{rk'_n Wh^2}{4(2rL_1 + (r+1)\,L_2)} + \frac{k'_n Wh^2}{4(2rL_1 + (r+1)\,L_2)} = P(1+r)$$

Finally,

$$F_2 = \left[\frac{k'_n Wh^2(1+r)}{4(2rL_1 + (r+1)\,L_2)}\right]$$

and

$$M_{a2_{L2}} = F_2 \frac{L_2}{2} = \left[\frac{k'_n Wh^2(1+r)}{4(2rL_1 + (r+1)\,L_2)}\right]\frac{L_2}{2}$$

$$M_{a2_{L2}} = \left[\frac{k'_n Wh^2(1+r)L_2}{8(2rL_1+(r+1)\,L_2)}\right] \qquad \text{.....(7c)}$$

The equilibrium of all moments at joint a_2 is given by

$M_{a2_h} = M_{a2_{L1}} + M_{a2_{L2}}$

$M_{a2_{L1}} = \left[\frac{r\,k'_n\,L_1\,Wh^2}{8(2rL_1+(r+1)\,L_2)}\right]$ and $M_{a2_{L2}} = \left[\frac{k'_n Wh^2(1+r)L_2}{8(2rL_1+(r+1)\,L_2)}\right]$

$$M_{a2_h} = \left\{\frac{k'_n Wh^2}{8(2rL_1+(r+1)\,L_2)}[rL_1 + (1+r)L_2]\right\} = \left[\frac{r\,k'_n\,L_1\,Wh^2}{8(2rL_1+(r+1)\,L_2)} + \frac{k'_n Wh^2(1+r)L_2}{8(2rL_1+(r+1)\,L_2)}\right]$$

$$M_{a2_h} = \left\{\frac{k'_n Wh^2}{8(2rL_1+(r+1)\,L_2)}[rL_1 + (1+r)L_2]\right\} = \left[\frac{k'_n Wh^2}{8(2rL_1+(r+1)\,L_2)}[rL_1 + (1+r)L_2]\right]; \qquad \text{Okay}$$

Generalised Equations for a Determination of Moment and Forces at n^{th} Story Level

Determination of axial forces at the internal column for the second floor (2^{nd}) from ground level (i.e. P_{2n})

$$P_{2n} = \frac{\left[(n-1)+2\sum_{i=1}^{\lambda}(n-\lambda)\right]kWh^2}{4(2rL_1+(r+1)\,L_2)} \qquad \text{.....(8)}$$

Total number of storeys n = 10

Number of storeys j (i.e. n-i) above level i= 2;

j = n - i = 10 - 2 = 8

The last odd number **λ** consider

λ = (ij + 1)= (2 x 8 +1) = 17

$$\left[(n-1)+2\sum_{i=1}^{\lambda}(n-\lambda)\right] \qquad \text{……(8a)}$$

$$=(n-1)+2(n-3)+2(n-5)+2(n-7)+2(n-9)+2(n-11)+2(n-13)$$
$$+2(n-15)+2(n-17)$$

$=$ 17n-161

Alternatively, this can be similarly computed using the following procedures

Sum of all odd numbers = -2j(j+2)-1= -2x8(8+2) -1= -16(10) -1= -161

Sum of all n = ij+1 = 2x8+1 = 16+1 = 17n …..(8b)

Therefore, equation (8) can be re-written as (9)

$$P_j = \frac{[n(ij+1)-\{(2j(j+2))+1\}]kWh^2}{4(2rL_1+(r+1)\,L_2)} = \frac{[n(2x8+1)-\{(2x8(8+2))+1)\}]kWh^2}{4(2rL_1+(r+1)\,L_2)}$$

$$P_j = \frac{[17n-161]kWh^2}{4(2rL_1+(r+1)\,L_2)} = \frac{\beta_i W_j h^2}{4(2rL_1+(r+1)\,L_2)} \qquad \text{…..(9)}$$

If $\beta_n = 1+2(nj)$

and since $j = n-1;$

Therefore, let

$\beta_n = 1+2(nj)$ or $\beta_n = 1+2n(n-1)$ or $\beta_n = 2n^2-2n+1$ …..(9a)

$$W_n = \left(W_{min}+\left(\frac{n+3}{n}\right)K_{max}\right)b = 0.1042bW_{bas}\left(\frac{2n-3}{n-3}\right) \;;\; \forall, n > 3$$

$$a_n = \frac{h^2}{4[2rL_1+(1+r)L_2]} \qquad \text{…..(9b)}$$

From Figure 4.0a, the axial load, P_{1n}at the left and right exterior columns is given by

$$P_{1n} = \frac{r[2n^2-2n+1]\left[\left(W_{min}+\left(\frac{n+3}{n}\right)K_{max}\right)b\right]h^2}{4[2rL_1+(r+1)\,L_2]} \qquad \text{…..(9c)}$$

and the axial load,P_{2n}at the left and right interior columns is given by

$$P_{2n} = \frac{\{1+2n(n-1)\}\left[\left(W_{min}+\left(\frac{n+3}{n}\right)K_{max}\right)b\right]h^2}{4(2rL_1+(r+1)\,L_2)}$$

$$P_{2n} = \frac{[2n^2-2n+1]\left[\left(W_{min}+\left(\frac{n+3}{n}\right)K_{max}\right)b\right]h^2}{4[2rL_1+(r+1)\,L_2]} \qquad \text{…..(9d)}$$

From Figure 4.0 (a), all these values of P_{1n} and P_{2n} (in 9c & 9d) can be re-written for the n^{th} storey counted from the topmost floor as equations (9e and 9f)

$$P_{1n} = \frac{r\,\beta_n W_n h^2}{4(2rL_1+(r+1)\,L_2)} = r\,a_n\beta_n W_n \qquad \text{…..(9e)}$$

$$P_{2n} = \frac{\beta_n W_n h^2}{4(2rL_1+(r+1)\,L_2)} = a_n\beta_n W_n \qquad \text{…..(9f)}$$

Shearing forces H_{1n} and H_{2n}along the columns are

$$H_{1n} = \frac{(2n-1)\, r\, W_n h^2}{4(2rL_1+(r+1)\, L_2)} = \frac{r\,(2n-1)\, a_n W_n L_1}{h} \quad \text{.....(10a)}$$

$$H_{2n} = \frac{(2n-1) W_n h^2 [rL_1+(1+r)L_2]}{4(2rL_1+(r+1)\, L_2)} = \frac{a_n W_n (2n-1)[rL_1+(1+r)L_2]}{h} \quad \text{.....(10b)}$$

Shearing forcesF_{1n} and F_{2n}along the beams are

$$F_{1n} = \frac{(n-1)\, r\, W_n h^2}{4(2rL_1+(r+1)\, L_2)} = (n-1)4ra_n W_n \quad \text{.....(11a)}$$

$$F_{2n} = \frac{(n-1)(r+1)\, W_n h^2}{4(2rL_1+(r+1)\, L_2)} = (n-1)(r+1)4a_n W_n \quad \text{.....(11b)}$$

Bending moments along the beams and columns shown in Figure 4.0(b),

The moment at underside (i.e. bottom) of an exterior rigid joint of a column for a particular storey level, n is M^B_{1nc}

$$M^B_{1nc} = H_{1n}\frac{h}{2} = \frac{r\,(2n-1)\, a_n W_n L_1}{2} \quad \text{.....(12a)}$$

The moment at topside (i.e. top) of an exterior rigid joint of a column for a particular storey level, n is M^B_{1nc}

$$M^T_{1nc} = H_{1n-1}\frac{h}{2} = \frac{r\,(2n-1)\, a_n W_n L_1}{2} \quad \text{.....(12b)}$$

The moment along the exterior rigid joint of a beam for a particular storey level, n isM_{1nb}

$$M_{1nb} = F_{1n}\frac{L_1}{2} = 2r(n-1)a_n W_n L_1 \quad \text{.....(12c)}$$

But, when n=1,the moment along the exterior beam at the topmost floor M_{11b} is given by

$$M_{11b} = F_{11}\frac{L_1}{2} = \frac{r}{2}\, a_n W_n L_1 \quad \text{.....(12c)}$$

The moment at underside (i.e. bottom) of an interior rigid joint of a column for a particular storey level, n is M^B_{2nc}

$$M^B_{2nc} = H_{2n}\frac{h}{2} = \frac{a_n W_n (2n-1)[rL_1+(1+r)L_2]}{2} \quad \text{.....(12d)}$$

The moment at topside (i.e. top) of an exterior rigid joint of a column for a particular storey level, n is M^T_{2nc}

$$M^T_{2nc} = H_{2n-1}\frac{h}{2} = \frac{a_n W_n (2n-1)[rL_1+(1+r)L_2]}{2} \quad \text{.....(12e)}$$

The moment along the interior rigid joint of a beam for a particular storey level, n isM_{2nb}

$$M_{2nb} = M^R_{2nb} = F_{2n}\frac{L_2}{2} = 2(n-1)(r+1)a_n W_n L_2 \quad \text{.....(12d)}$$

Analytical Examples

A 14m three bays (L_1=6m, L_2 =4m) framed medium rise 48m (h=3m), 16 storey building is 30m long with 6m columns or frames spacing (i.e.six transverse braced frames).Basic wind speed for Maiduguri where the building is sited is W_{bas} =47 ms^{-1}; altitude factor S_a = 1.34 for an elevation of 340m above the sea level and the following relevant factors: permanent seasonal S_p = 1.0; directionality S_d = 0.99, dynamic K_b = 1.0;dynamic augmentation C_r =0.04; size effect C_a = 0.83; external surface pressure C_{pe} = 1.1; and terrain factor varying between 1.362 at 10m and 1.900 at 48m above ground level. The conversion factor of a triangular pressure to evenly distribution pressure is 0.625. Using the appropriate equations and formulae determine the axial, shearing forces and their corresponding bending moments on the columns and beams; at the sixth floor of a rigid unbraced frame subjected mainly to aerodynamic loadings.

Solution

From equation 2f, the equivalent uniformly distributed load from a W_{bas} = 47 m/s applied at each floor level is

$$W_n = W_{max} = \left(W_{min} + \frac{n+3}{n}K_{max}\right)b = 0.1042bW_{bas}\left(\frac{2n-3}{n-3}\right)\ ;\ \forall, n > 3$$

$$W_n = 0.1042x6x47x\left(\frac{2x16-3}{16-3}\right) = 65.53 \text{ kN}$$

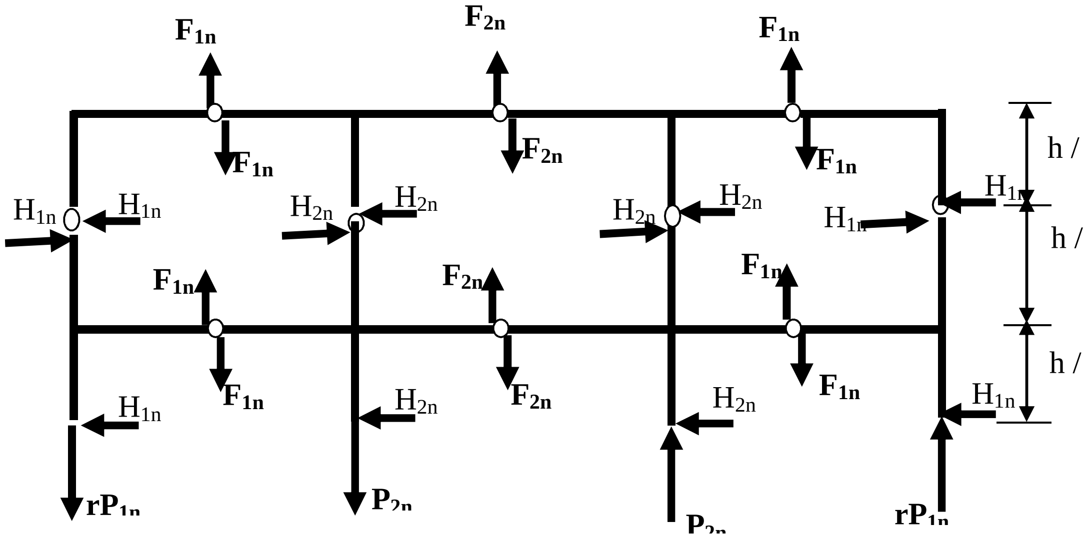


Figure 4.0 (a) : The Axial and Shearing forces along the Columns and

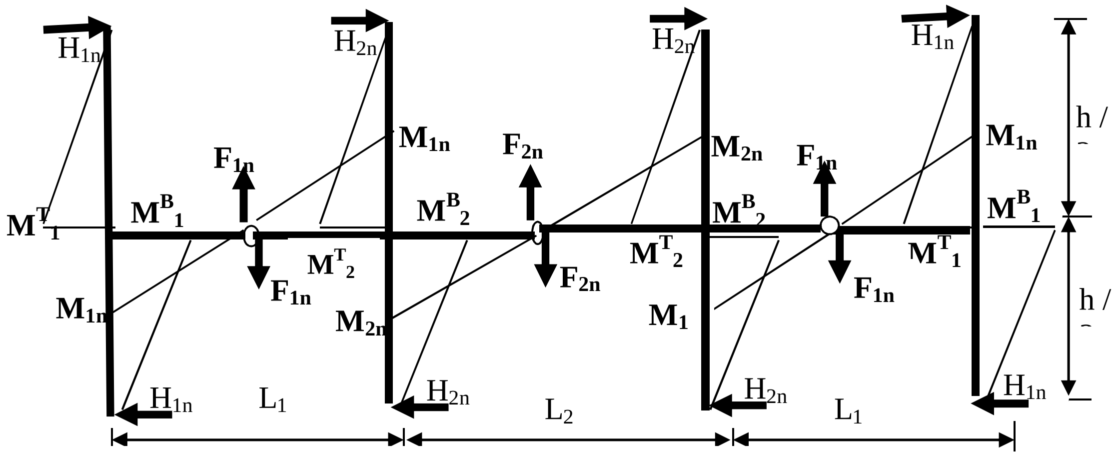


Figure 4.0 (b) : The Bending Moments on the Columns and Beams

Therefore, column forces and moment are determined as follows

a.) The Exterior, P_{1n} and Interior, P_{2n} Column Axial Forces

From equations 9(a and b), the non-dimensional storey number β_n and variable dimensions, a_n influence coefficients are determined.

$$\beta_n = 2n^2 - 2n + 1 = 2x16^2x2 - 16 + 1 = 481$$

$$a_n = \frac{h^2}{4[2rL_1 + (1+r)L_2]} = \frac{3^2}{4[2x4x6 + (1+4)4]} = 0.03309$$

From equations 9(c, d and f)

$$P_{1n} = \frac{r\,\beta_n W_n h^2}{4(2rL_1 + (r+1)L_2)} = r\,a_n\beta_n W_n = 4x0.03309x481x65.53 = 4172\text{ kN}$$

$$= \frac{\beta_n W_n h^2}{4(2rL_1 + (r+1)\,L_2)} = a_n\beta_n W_n = 0.03309x481x65.53 = 1043\text{ kN}$$

b.) The Exterior, H_{1n} and Interior, H_{2n} Shearing Forces along the Columns

From equations 10(a and b),

$$H_{1n} = \frac{(2n-1)\,r\,W_n h^2}{4(2rL_1+(r+1)\,L_2)} = \frac{r\,(2n-1)\,a_n W_n L_1}{h} = \frac{4x0.03309x65.53x6x(2x16-1)}{3} = 538\text{ kN}$$

$$H_{2n} = \frac{(2n-1)W_n h^2[rL_1 + (1+r)L_2]}{4(2rL_1 + (r+1)\,L_2)} = \frac{a_n W_n(2n-1)[rL_1 + (1+r)L_2]}{h}$$

$$H_{2n} = \frac{4x0{,}03309x65.53(2x16-1)x[4x6+(1+4)4]}{3} = \frac{8.674x31x44}{3} = 986\text{ kN}$$

c.) The Exterior, F_{1n} and Interior, F_{2n} Shearing Forces along the Beams

From equations 11(a and b),

Shearing forces F_{1n} and F_{2n} along the beams are

$$F_{1n} = \frac{(n-1)\,r\,W_n h^2}{4(2rL_1+(r+1)\,L_2)} = (n-1)4ra_nW_n = (16-1)x4x4x0.03309x65.53 = 520.41\text{ kN}$$

$$F_{2n} = \frac{(n-1)(r+1)W_n h^2}{4(2rL_1 + (r+1)L_2)} = (n-1)(r+1)4a_nW_n$$

$$F_{2n} = (16-1)(4+1)x4x0.03309x65.53 = 650.52\text{ kN}$$

d.) Bending Moments along the Beams and Columns

From equations 12(a - d)

The moment at underside (i.e. 16th Floor) of an exterior rigid joint of a column for a particular storey level, n is M^B_{1nc}

$$M^B_{1nc} = H_{1n}\frac{h}{2} = \frac{r\;a_n W_n L_1(2n-1)}{2} = \frac{4x0.03309x65.53x6x(2x16-1)}{2} = 807\text{ kNm};\ (\text{i.e.}\,H_1 = 538x1.5)$$

The moment at topside (i.e.15th Floor) of an exterior rigid joint of a column for a particular storey level, n is M^B_{1nc}

$$M^T_{1nc} = H_{1n-1}\frac{h}{2} = \frac{r\;a_n W_n L_1(2n-1)}{2} = \frac{4x0.03309x65.53x6x(2x15-1)}{2} = 754\text{kNm}$$

The moment along the exterior rigid joint of a beamfor a particular storey level, n is M_{1nb}

$$M_{1nb} = F_{1n}\frac{L_1}{2} = 2ra_nW_nL_1(n-1)$$

$$M_{1nb} = 2x4x0.03309x65.53x6x(16-1) = 1561\text{ kNm (i.e.}\,520.41\text{ x3)}$$

But, when n=1,the moment along the exterior beam at the topmost floor M_{11b} is given by

$M_{11b} = F_{11}\frac{L_1}{2} = \frac{r}{2}a_nW_nL_1 \quad = \frac{4}{2}x0.03309x65.53x6 = 26\,kNm$

The moment at underside (i.e. 16th Floor) of an interior rigid joint of a column for a particular storey level, n is M^B_{2nc}
$M^B_{2nc} = H_{2n}\frac{h}{2} = \frac{a_nW_n(2n-1)[rL_1+(1+r)L_2]}{2} = \frac{0.03309x65.53x(31)x[24+20]}{2} = 1479\ kNm\ (i.e.\ 986x1.5)$

The moment at topside (i.e. 15th Floor) of an exterior rigid joint of a column for a particular storey level, n is M^T_{2nc}

$$M^T_{2nc} = H_{2n-1}\frac{h}{2} = \frac{a_nW_n(2n-1)[rL_1+(1+r)L_2]}{2}$$

$M^T_{2nc} = \frac{0.03309x65.53(29)[44]}{2} = 1383kNm\ (i.e.\ 922x1.5)$

The moment along the interior rigid joint of a beam for a particular storey level, n is M_{2nb}

$$M_{2nb} = M^R_{2nb} = F_{2n}\frac{L_2}{2} = 2(n-1)(r+1)a_nW_nL_2$$

$M_{2nb} = 2x(15)(5)x0.03309x65.53x4 = 1301\ kNm\ (i.e.\ 650.52x2)$

e.) Equilibrium of Joints

From Figure 4b; in complying with the requirements of the principle of minimum potential energy at all joints, the joint moment equilibrium is given as follows.
For the exterior columns,

$$M^T_{1nc} + M^B_{1nc} + M_{1nb} = 807 + 754 - 1561 = 0$$

Similarly, for the interior columns

$$M_{1nb} + M_{2nb} - M^T_{2bc} - M^B_{2nc} = 1561 + 1301 - 1383 - 1479 = 2862 - 2862 = 0$$

Similar equilibrium is obtained from sum horizontal and vertical force for the substitute frames.

DISCUSSION OF RESULTS

The slab is considered to be an infinitely rigid structural element along the plane of the frame; its rigidity assists in the distribution of the horizontal load from wind or seismic loadings to be concentrated not within the inter storey column height but at the levels of the slab which acts as a non-deformable tie to unite all the frames together with any available primary and secondary beams. Although, for most practical situations, the need for beams and columns are usually dictated by the architectural and technological considerations, but for structural purposes, the sizes of reinforced concrete beams in millimeters are to be determined by equation 13.

$$h_b = 1.45\sqrt{\frac{M_0}{b\,f_{ck}}} \qquad \ldots 13$$

where; M_o is the bending moment at the beam joint for wind or seismic loadings or mid span bending moment for a simply supported beam from vertically acting dead and imposed loadings; $b \geq \frac{L}{60}$ is the concrete web, L is the beam effective span and f_{ck} characteristic strength of the concrete. Depending on the percentage of reinforcement required, the required area of concrete column may be determined by equations 14(a and b)

$A_c = \frac{3\ N}{f_{ck}}$ for $N \leq$ 1000kN …14a

$A_c = \frac{2.25\ N}{f_{ck}}$ for N > 1000kN but $Ac \geq 122{,}500\ mm^2$ …14b

From the bending moment of 1479 kNm obtained at the 16th floor counted from the top, assuming a beam web b_w =300mm and grade 25 concrete, from equation (13), a 650mm overall depth h of the beam may be required. Similarly, since the exterior column gave an axial force of 4172 kN, eq. (14b) shows the area of appropriate concrete concrete is 375480mm². This could squared column of 625mm or 500x750mm respectively. This approach and

values offer useful preliminary criteria for preliminary sizing of beams and columns for rigid frames subjected to aerodynamic loadings.

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